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A note on liquid vortex sloshing and Kelvin's equilibria

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Observations of liquid vortex sloshing and Kelvin's equilibrium states were made inside a cylindrical container using a spinning disk near its base. Both steady and periodic free-surface sloshing phenomena were found to take place. During periodic sloshing, the air core sustained shape transformations, assuming an elliptical cross-section at the end, and then collapsed forming a pair of vortices. Kelvin's equilibrium states emerged at lower liquid levels. These were stable within an interval of rotational speeds. The bandwidth of stationary states decreased as the wavenumber (N) increased. For N greater than six, the states appeared critically stable. Between equilibria, unstable transitional regions were found to exist. As the liquid level was decreased, the core shape spectrum shifted towards smaller frequencies.

1. Introduction

Vortices control to a large extent mass, momentum and energy transport in nature and technology. The core, the most active part of every vortex, is known to influence dramatically and even dictate the evolution and further behaviour of the host flow field. This profound activity of the vortex centre has been attributed to its ability to act as a wave-guide to several types of disturbances. In some flow situations, such as in the intakes of liquid pumps and draft tubes of water turbines, disturbances originating from the core produce vibrations and noise (Escudier 1987; Knauss 1987). In tornadoes the variations of swirl due to waves near the ground enhance their destructive character (Maxworthy 1972). The existence of waves inside the core was first predicted by Kelvin (1880) in his attempt to develop an atomic theory for the molecular structure of matter. Although his vortex theory of matter has long been abandoned, elements are used to interpret phenomena in low-temperature physics, (Feymman 1954; Yarmchuk, Gordon & Packard 1979).

In the present note we are interested in the types of wave that propagate around the core disturbing the circularity of the cross-section. The events where vortex liquid sloshing is taking place will be described, continuing with the special class of steadily rotating vortices, which correspond to standing waves when viewed from a rotating frame of reference.

2. Laboratory experiments

2.1. *Apparatus*

Free-surface liquid sloshing and vortex equilibria were observed in a stationary cylindrical container, with a flat disk rotating in the counter-clockwise direction near the bottom. The experimental facility is shown schematically in figure 1. The rotary

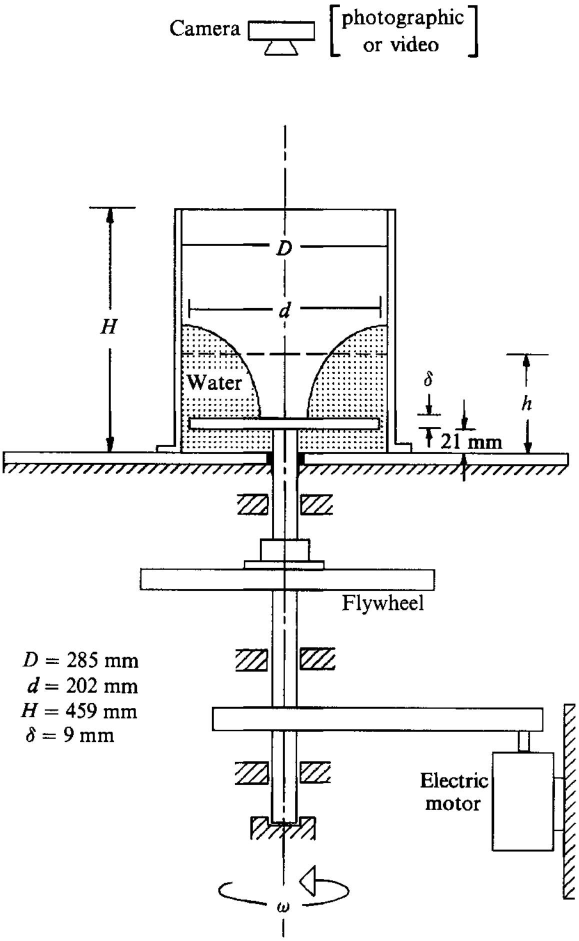


FIGURE 1. A schematic of the apparatus.

motion imparted to the fluid by the disk generates a centrifugal force field which compels the liquid towards the circumferential wall. The receding water exposed part of the surface of the disk to air, whereby the line of intersection between the surfaces of the solid disk and the liquid outlined Kelvin's static shapes of the core. In order to bring into relief the various patterns, the liquid was coloured with a blue water-soluble dye. An electronic variable-speed controller allowed speed selections up to 1200 revolutions per minute (r.p.m.). In order to improve the constancy of the rotary disk motion an aluminium flywheel was incorporated. A stroboscope was used to obtain the r.p.m. values and to freeze the distinct shapes in time. A video and a photographic camera, placed above the Plexiglas container, were used to record the patterns. The initial water level was measured using a ruler placed on the side of the tank.

2.2. *Experimental observations*

The experiments described in the present note, began when I became interested in the problem of particle concentrations in combined liquid vortices, produced within a stationary cylindrical container, using revolving disks and cylinders (Vatistas 1989). During a detailed mapping of the steady free-surface profile as a function of the disk diameter (d), the original water level (h) and the rotational speed of the disk

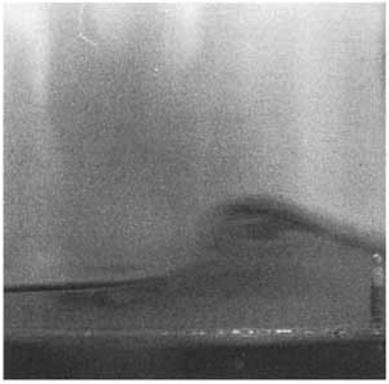


FIGURE 2

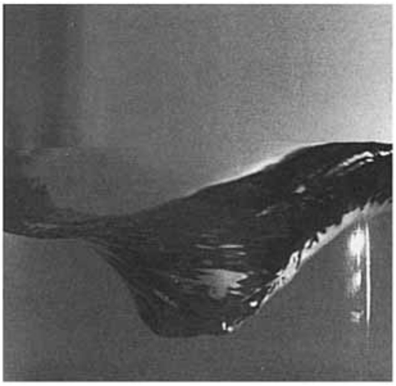
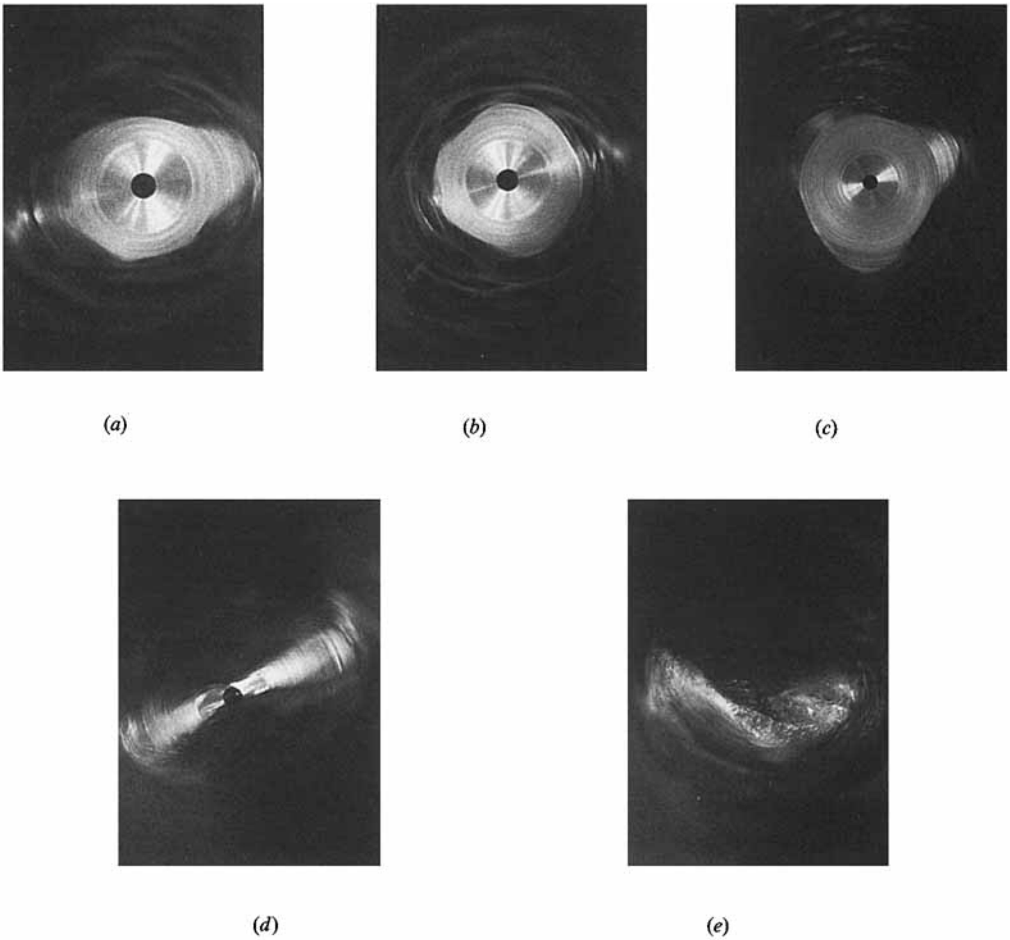


FIGURE 3

FIGURE 2. Solitary wave generated during liquid draining from a cylindrical reservoir.

FIGURE 3. Liquid free-surface undulations during sloshing with $\omega = 240$ r.p.m., $h = 150$ mm and $d = 252$ mm.



(*a*) (*b*) (*c*)

(*d*) (*e*)

FIGURE 4. Contours of the air core during periodic sloshing with $\omega = 780$ r.p.m., $h = 293$ mm and $d = 252$ mm. Time increases from (*a*) to (*e*).

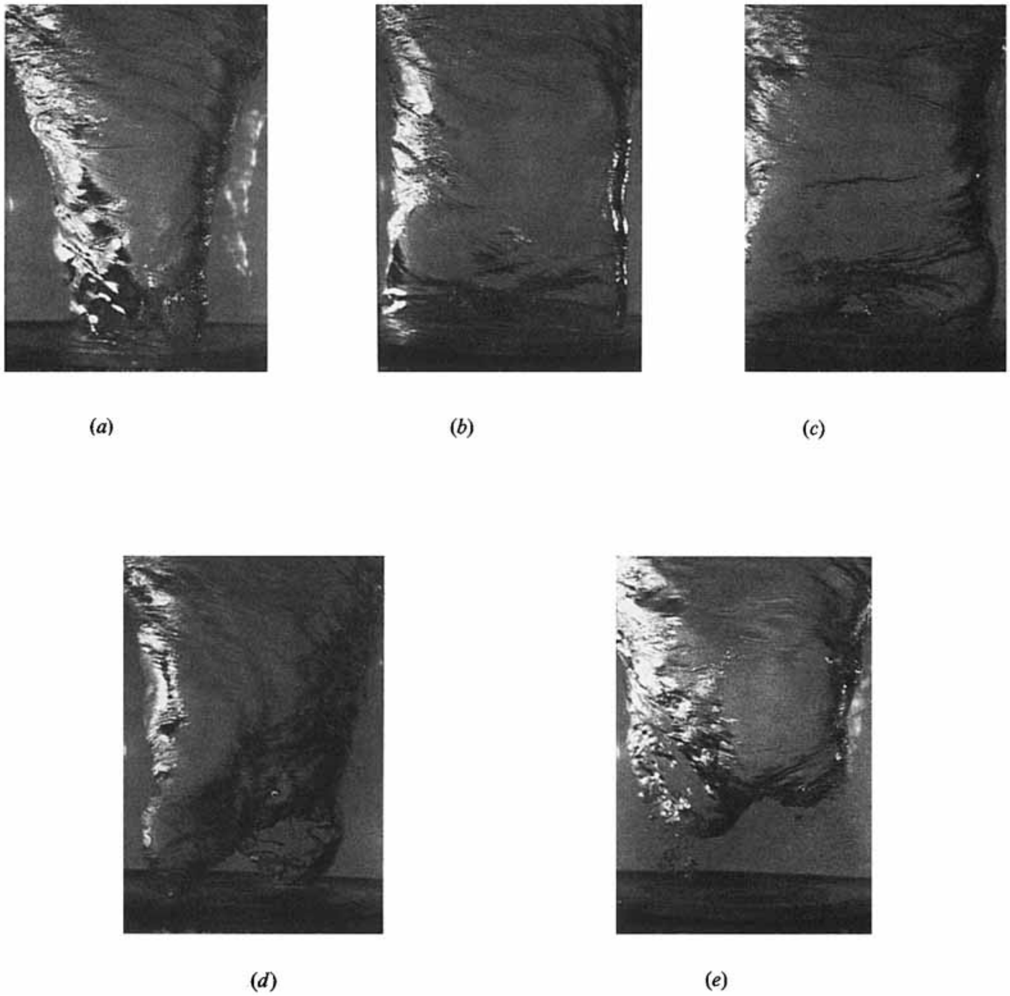

(a) (b) (c) (d) (e)

FIGURE 5. Side view of the air core during periodic sloshing with $\omega = 780$ r.p.m., $h = 293$ mm and $d = 252$ mm. Time increases from (a) to (e).

(ω), several problems were encountered. All these were attributed to the dynamic nature of the core. For vortices with tall thin cores, the familiar axisymmetric and bending waves described by Maxworthy, Hopfinger & Redekopp (1985) forced the vortex centre to precess and undulate. In addition to these problems, however, steady or periodic sloshing of the free liquid surface occurred. Similar, but moderate, steady free-surface undulations were also observed during liquid drainage from a cylindrical tank with a diameter equal to 285 mm, and an initial water level height approximately equal to 330 mm. For a cylindrical concentric exit port diameter and length equal to 31.75 mm and 165 mm respectively, the shallow water solitary wave of figure 2 was generated as soon as the water level dropped to approximately 30 mm. The strength of the wave was found to depend on the initial vorticity of the liquid. Dye injection in the centre during periodic sloshing in the tank–disk experiment, showed that the core was undergoing a dramatic transformation in time. For example when $\omega = 240$ r.p.m. $h = 150$ mm and $d = 252$ mm, the periodic sloshing shown in figure 3 was observed. Initially, the free surface was relatively calm,

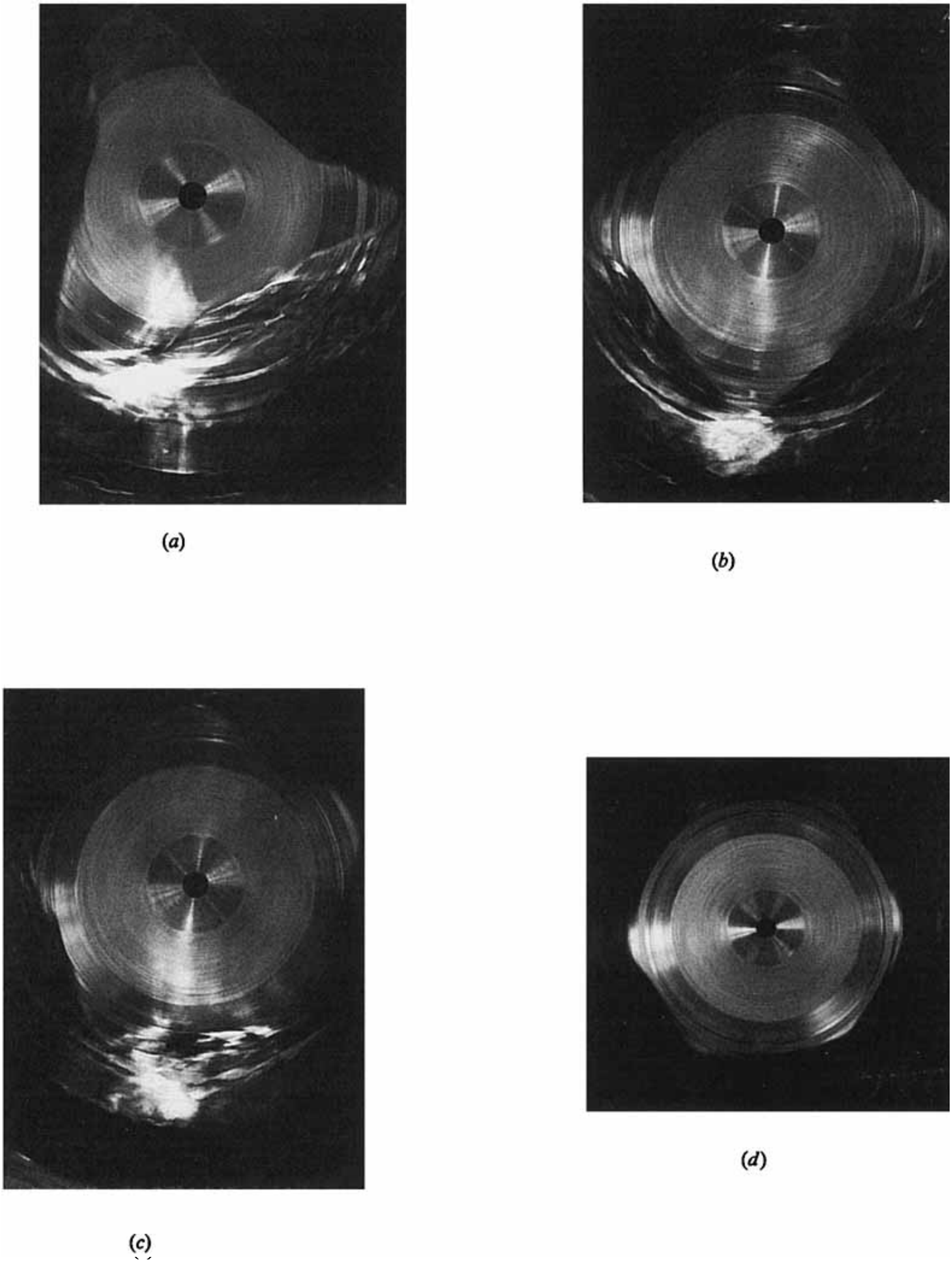


FIGURE 6. Vortex cores under the influence of different numbers of standing waves: (*a*) $N = 3$, (*b*) $N = 4$, (*c*) $N = 5$ and (*d*) $N = 6$.

acquiring the shape of an inverted bell. The cross-section of this surface was almost circular in every elevation. Following the latter state, a good portion of the lower part of the core transformed slowly into an elliptical cross-sectional shape, and the liquid surface began to undulate. The minor axis of the ellipse was then shortened

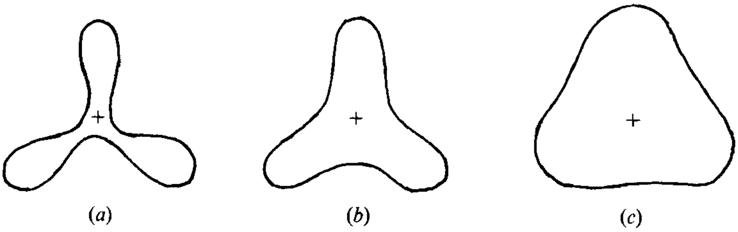


FIGURE 7. Core transformations with $N = 3$, for (a) $\omega_{i,3}$, (b) $\omega_{i,3} < \omega < \omega_{f,3}$ and (c) $\omega_{f,3}$.

and the major one was elongated while the sloshing behaviour intensified. Finally the core collapsed inwards, the vortex was destroyed and the surface returned to its initial calm condition. The phenomenon was repeated at regular intervals with an average period of 65 s.

Under the above-mentioned conditions, we were only able to visualize the gross core-shape transformations. In our attempt to see details we searched and found a set of parameters ($\omega = 780$ r.p.m., $h = 293$ mm and $d = 252$ mm) where a periodic sloshing was present and the air cavity exposed part of the disk surface. In the beginning, a downward developing bell-like air cavity was observed. As soon as its vertex touched the disk and exposed part of its surface, the core started to form the various shapes shown in figure 4(*a–e*). Waves generated at the bottom were convected towards the top causing the free surface to slosh. As in the previous case, the core assumed the elongated elliptical cross-section of figure 5(*a, b*), broke into two vortices, which are depicted in figures 4(*d*) and 5(*c, d*), and then was destroyed, see figures 4(*e*) and 5(*e*). It is worth noting in passing the striking resemblance of figure 5(*d*) with the twin tornado of Elkhard, Indiana, on Palm Sunday in 1965, see Ward (1972). During visual inspections while the experiment was in progress, as well as viewing the videotapes in slow motion afterwards, we noticed that some of the convected waves at higher elevations were interfering visually with the shapes formed on the disk surface. Reducing the water level to alleviate the latter interference, Rankine's static shapes emerged.

Vortex equilibrium patterns were clearly observed mainly on the exposed part of the disk. A sample of these is shown in figure 6(*a–d*). The experimentally obtained air core shapes are remarkably similar to the contours predicted by the theoretical studies of Kozlov & Makarov (1985), Helfrich & Send (1988) and others. These are also similar to the shapes formed by evaporating liquid drops reported by Holter & Glasscock (1952) and to cylindrical thin-shell vibrations presented in Markus (1988). The shapes were exceptionally stable. If perturbed or even completely destroyed by the insertion of an obstruction inside the flow, they would reappear in their original shape. Each vortex equilibrium state could be reached by approaching ω_N either from the left or from the right. The basic shape was retained by the core for ω values within the interval $[\omega_{i,N}, \omega_{f,N}]$. Nonetheless, as shown in figure 7, there were differences. For example, when N was equal to three, the core was modulated by the presence of the three standing waves within the interval $[\omega_{i,3}, \omega_{f,3}]$. However, their amplitude decreased as the speed of the disk increased. By immobilizing the shapes, with the use of a stroboscope, several small travelling disturbances were seen to encircle the periphery. As expected, the rotational speed of the frame containing the basic static pattern was found to lag behind the speed of the disk. It is very clear from figure 8 that the band-width $[\omega_{i,N}, \omega_{f,N}]$ decreased as N increased. For $N > 6$ the states appeared critically stable. These were so closely populated, that a small

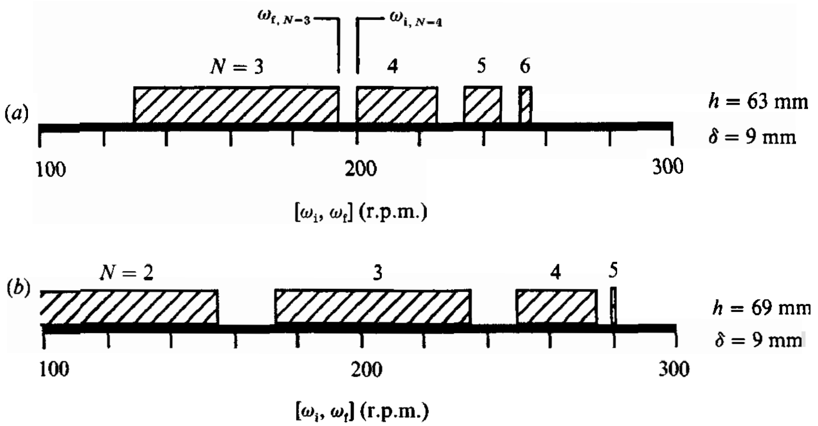


FIGURE 8. Spectra of stable and transitional equilibrium states.

increase of ω did not necessarily yield the next higher stationary state. In addition to the latter problem, the surface tension excessively rounded off the crest and troughs of the waves, thus prohibiting the clear identification of high-wavenumber core patterns. The existing apparatus was undoubtedly not suitable for the study of N beyond six. For higher liquid levels, the various shapes emerged at higher ω values. In the proximity of $\omega_{f,N}$, strong waves encircled the perimeter of the core giving the impression that the equilibrium patterns were vibrating in the azimuthal direction. For vortices with speeds lying between consecutive equilibrium states, $[\omega_{f,N}, \omega_{i,N+1}]$, the core shape was found to be in a transitional mode possessing a distorted, unstable configuration of the higher state $(\omega_{i,N+1})$. During elevation from one equilibrium state $\omega_{f,N}$ to the next $\omega_{i,N+1}$, the unsteady core generated distorted shapes of the $N+1$ state, stabilizing in a short period of time to the $N+1$ configuration. The roughness and low-level wobbling of the disk, did not appear to have any appreciable influence on the above-mentioned phenomena.

The purpose of the present note was to report on the existence, and give a description, of the phenomena involved in a qualitative manner. There is, however, no doubt that more controlled experiments with the aim of accurately quantifying the processes are required.

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